THERMAL NOISE MEASUREMENTS

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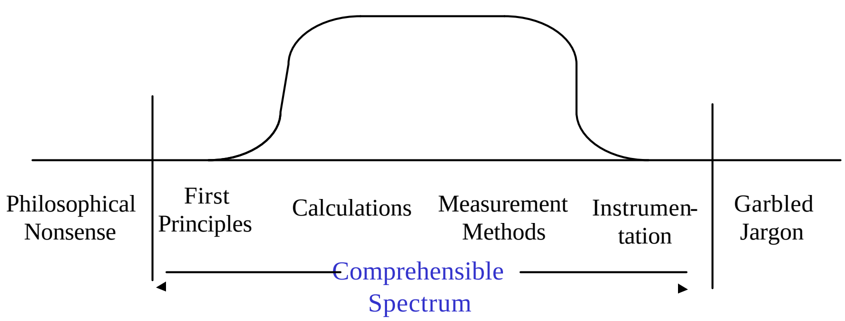
INTRODUCTION

- General Content:



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    - Noise Temperature Definition
    - Microwave Networks & Noise
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    - Total-power radiometer
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      - simple, idealized case
      - not so simple case
    - Uncertainties
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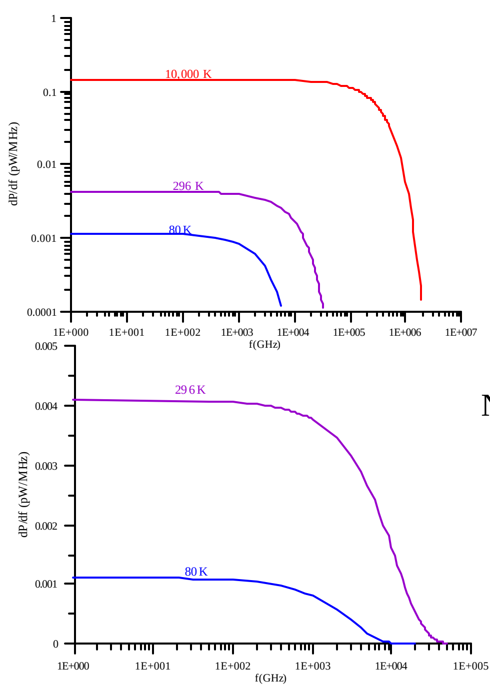
I. BASICS

Nyquist Theorem

- Derivation:
  - Electr. Eng. [1-4]
  - Physics, Stat. Mech. [4]
- For passive device, at physical temperature T, with small)*f*,

$$\langle P_{avail}(f)\rangle = \frac{hf}{e^{hf/(k_B T)}-1}\Delta f$$



$$\langle P_{avail}(f)\rangle = \frac{hf}{e^{hf/(k_B T)}-1}\Delta f$$

Note: very small powers.

- Limits
  - small f : $<P_{avail}>$. $k_B T$) f [1 – hf/(2$k_B T$)]
    . $k_B T$) f
  - large f : ÿ 0
  - knee occurs around f(GHz) . 20 T(K)
- Quantum effect
  - h/k_B = 0.04799 K/GHz
  - So at 290 K, 1 % effect at 116 GHz
    at 100 K, 1 % effect at 40 GHz
    at 100 K, 0.1 % effect at 4 GHz
    30 K @ 40 GHz ÿ 6.4%, 0.26 dB

NOISE TEMPERATURE

- What about active devices? Can we define a noise temperature?
- Several different definitions used:
  - delivered vs. available power
  - with or without quantum effect
    i.e., does T_{noise} - P_{avail} or is T_{noise} the physical temperature that would result in that value of P_{avail} ?

- IEEE [5]: "(1)(general)(at a pair of terminals and at a specifice frequency) the temperature of a passive system having an available noise power per unit bandwidth equal to that of the actual terminals."
  and
  "(4)(at a port and at a selected frequency) A temperature given by the exchangeable noise-power density divided by Boltzmann's constant, at a given port and at a stated frequency."

- We (I) will use second definition, noise temp / available noise-power density divided by Boltzmann's constant.
- It is the common choice in international comparisons [6] and elsewhere [7].
- It is much more convenient for amplifier noise considerations (at least for careful ones)

- So $P_{avail} = \mathrm{k_B} T_{noise})f$
- And for passive devices,

$$T_{noise} = \frac{1}{k_B}\left[\frac{hf}{e^{hf/(k_B T)} - 1}\right] \approx T_{phys}$$

- Convenient to define "Excess noise ratio"

$$ENR_{(avail)} \equiv \frac{T_{(avail)} - T_0}{T_0} \qquad T_0 = 290\,K$$

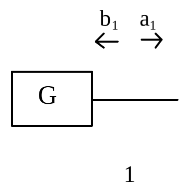
T=9500 K Y ENR – 15.02 dB

No matter what definition of noise temperature you choose,
it is helpful to state your choice.

MICROWAVE NETWORKS & NOISE [8,9]

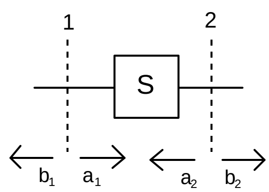
- Assume lossless lines, single mode.
- Travelling-wave amplitudes a, b.
- Normalized such that $P_{del} = |a|^2 - |b|^2$
- May be a little careless about B; assume that it's 1Hz where needed.

- Describe (linear) one-ports by



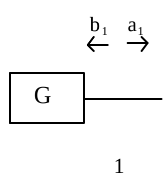
$$a_1 = \Gamma_G b_1 + \hat{a}_G$$

- And (linear) two-ports by



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

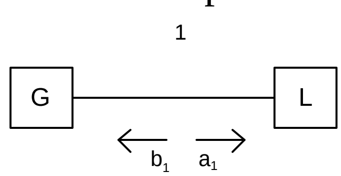
- Available power:



$$P_G^{(avail)} = \frac{|\hat{a}_G|^2}{1 - |\Gamma_G|^2}$$

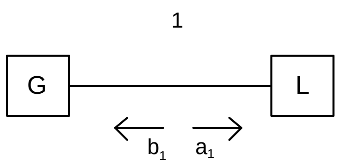
$$\langle |\hat{a}_G|^2 \rangle = (1 - |\Gamma_G|^2)\, k_B T_G$$

- Delivered power:



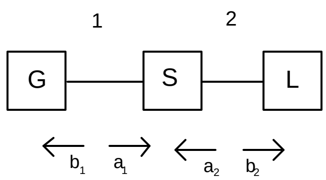
$$P_1^{del} = |a_1|^2 - |b_1|^2 = |a_1|^2 (1 - |\Gamma_L|^2)$$

- Mismatch factor: $M_1 \equiv p_{1,del}/p_{1,avail}$



$$M_1 = \frac{(1 - |\Gamma_L|^2)(1 - |\Gamma_G|^2)}{|1 - \Gamma_L \Gamma_G|^2}$$

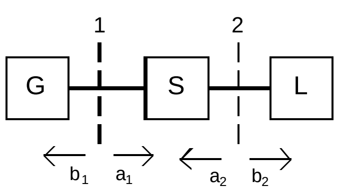
- Efficiency: $\eta_{21} \equiv p_{2,del}/p_{1,del}$



$$\eta_{21} = \frac{|S_{21}|^2 [1 - |\Gamma_L|^2]}{|1 - \Gamma_L S_{22}|^2 [1 - |\Gamma_{SL}|^2]}$$

$$\eta_{21} = \frac{|S_{21}|^2 [1 - |\Gamma_L|^2]}{|1 - S_{22}\Gamma_L|^2 - |(S_{12}S_{21} - S_{11}S_{22})\Gamma_L + S_{11}|^2}$$

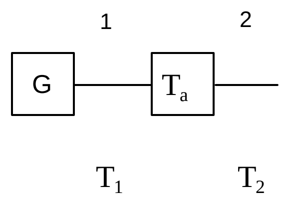
- Available power ratio:
  $\alpha_{21} \equiv p_{2,avail}/p_{1,avail}$ $(\hat{b}_1 , \hat{b}_2 = 0)$



$$\alpha_{21} = \frac{|S_{21}|^2 [1 - |\Gamma_G|^2]}{|1 - \Gamma_G S_{11}|^2 [1 - |\Gamma_{GS}|^2]}$$

$$\Gamma_{GS} = S_{22} + \frac{S_{12}S_{21}\Gamma_G}{1 - \Gamma_G S_{11}}$$

- For passive devices:



$$p_2^{avail} = \alpha_{21} p_1^{avail} + f_0(T_a)$$
$$T_2 = \alpha_{21} T_1 + f(T_a)$$

Say $T_1 = T_a$, then T_2 must $= T_a$, so

$$T_2 = T_a = \alpha_{21} T_a + f(T_a)$$
$$f(T_a) = (1 - \alpha_{21}) T_a$$

and therefore

$$T_2 = \alpha_{21} T_1 + (1 - \alpha_{21}) T_a$$

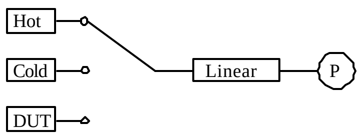
- Practical point: large vs. small T_1

II. NOISE-TEMPERATURE MEASUREMENT

Total-Power Radiometer [10-12]

- Two principal types of radiometer for noise-temperature measurements are Dicke radiometer and total-power radiometer [10].
- Total-power radiometer is most common for lab use, & that's what we'll discuss.

- Simple case: symmetric, matched (all ' ’s = 0)



Matched 6 $p_{del} = p_{avail}$

Linear 6 $P = a + bp_{del} = a + bp_{avail}$

2 standards (h,c) determine a, b:

$$a = P_c \,!\, bT_c \qquad Bk_B b = \frac{P_h - P_c}{T_h - T_c}$$

So $T_x = T_c + \frac{(Y_x - 1)}{(Y_h - 1)}(T_h - T_c)$, where $Y_x = \frac{P_x}{P_c}$, $Y_h = \frac{P_h}{P_c}$

- Not-so-simple case (unmatched, asymmetric)
  Three complications:
  - $p_{del} = Mp_{avail}$
  - $p_{del,rad} = 0\, p_{del,G}$, and $0_x \ldots 0_h \ldots 0_c$
  - $a, b = a('), b(')$

  – Handle first two by measuring and correcting.

– For dependence of *a* and *b* on ', have three choices:

- tune so that $'_h = '_c = '_x$ (very narrow frequency range, need special standards)
- characterize dependence on ' (broadband, but a lot of work, and difficult to get good accuracy)
- isolate (easy, accurate, but limits frequency range & difficult at low frequency)

– If isolate, *a* and *b* are (almost) independent of the source, and

$$T_x = T_{amb} + \frac{M_S h_S}{M_x h_x} \frac{(Y_x - 1)}{(Y_S - 1)} (T_S - T_{amb})$$

Uncertainties

- Simple case (matched):

$$T_x = T_a + \frac{(Y_x - 1)}{(Y_S - 1)}(T_S - T_a) \quad \frac{M_h ?_h}{M_x ?_x}$$

typically around 1 %

about 1 or 2%

small uncert,
but linearity
concern

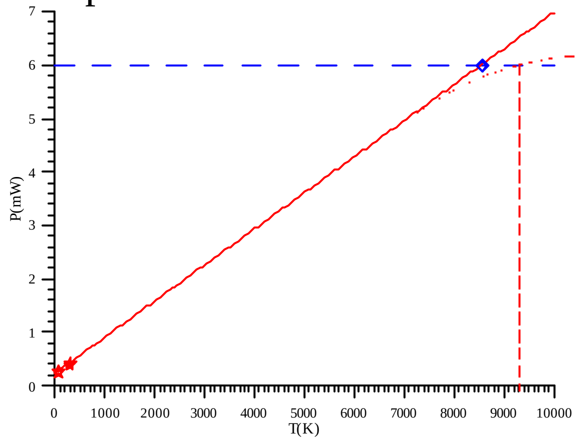
Uncert “should”
be negligible

- Simple-case uncerts (cont’d)
  - drift: temperature stability/control important
  - connector variability: hard to do much better than 0.1%, easy to do considerably worse.
  -) a,) b: depends on details of system, can make a crude estimate:

T_{rev} - T_e , *) ' *- 0.05 or 0.1

So) T_{in} - 0.05 or 0.1H T_e

- linearity: serious concern if T_x very different from standards, less (but some) worry if T_x near temperature of a standard.



- Uncertainties (more careful case)
  (Numbers are for NIST case) [13,14]
  - Radiometer equation:

$$T_x = T_{amb} + \frac{M_S h_S}{M_x h_x} \frac{(Y_x - 1)}{(Y_S - 1)} (T_S - T_{amb})$$

  - Ambient standard:

$$\frac{u_{T_x}(amb)}{T_x} = \left| \frac{T_x - T_S}{T_a - T_S} \right| \frac{T_a}{T_x} \, \text{õ}_{T_a}, \quad \text{õ}_{T_a} = \frac{0.1K}{296K} = 0.034\%$$

– “Other” standard:

$$\frac{u_{T_x}(S)}{T_x} = \left|1-\frac{T_a}{T_x}\right| \left|\frac{T_S}{T_a-T_S}\right| \frac{u_{T_S}}{T_S}, \quad \frac{u_{T_S}}{T_S} = 0.2\% (NIST\ W.G.),\ 0.8\% (NIST\ coax)$$

– Path asymmetry: (zero if connect to same port)

$$\frac{u_{T_x}(h/h)}{T_x} = \left|1-\frac{T_a}{T_x}\right| u_{h/h}, \quad u_{h/h} = 0.2\%\ to\ 0.56\%$$

– Mismatch:

$$\frac{u_{T_x}(M/M)}{T_x} = \left|1-\frac{T_a}{T_x}\right| u_{M/M}, \quad u_{M/M} \approx 0.2\%$$

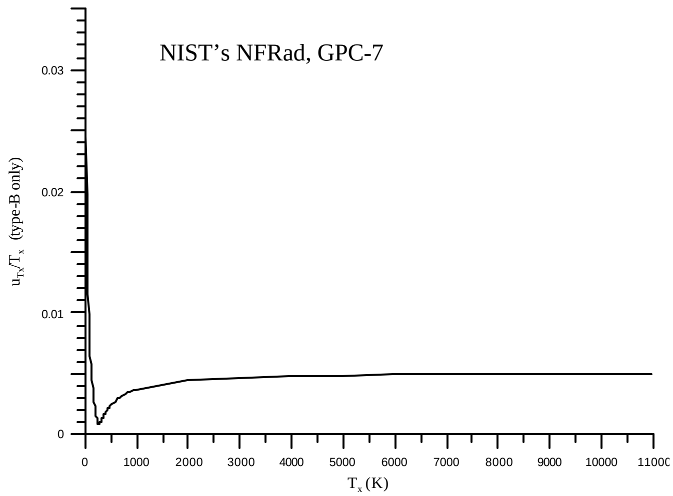
– Connectors:

$$\frac{u_{T_x}(conn)}{T_x} = u_0 \left|1-\frac{T_a}{T_x}\right| \sqrt{f(GHz)}, \quad u_0 \approx 0.053\%\ to\ 0.069\%$$

(depending on connector type)

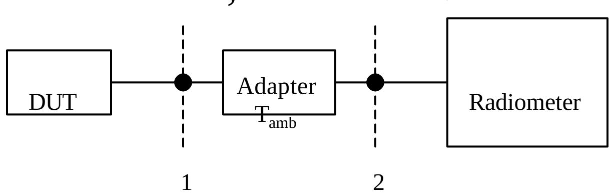
– Other: Nonlinearity, imperfect isolation, power ratio measurement, and broadband mismatch/frequency offset all lead to small (<0.1%) uncertainties for T_x around 10 000 K (for us/NIST).

- u_T(Type-B)/T as a function of T
  Standard relative uncertainty (1F)



Adapters

- Measure T at 2, want T at 1.



$$T_2 = \alpha_{21} T_{DUT} + (1-\alpha_{21}) T_{amb}$$

So

$$T_{DUT} = \frac{T_2 - (1-\alpha_{21}) T_{amb}}{\alpha_{21}}$$

III. NOISE FIGURE & PARAMETERS

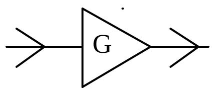
Noise Figure Defined

- Want a measure of how much noise an amplifier adds to a signal or how much it degrades the S/N ratio.

- Define Noise Figure, IEEE [15]:
  (at a given frequency) the ratio of total output noise power per unit bandwidth to the portion of the output noise power which is due to the input noise, evaluated for the case where the input noise power is $k_B T_0$, where $T_0 = 290$ K.
- Noise figure & signal to noise ratio[16]:

$$\frac{(S/N)_{in}}{(S/N)_{out}} = \frac{S_{in}/290\,K}{GS_{in}/(G\times 290\,K + N_{amp})} = \frac{G\times 290\,K + N_{amp}}{G\times 290\,K} = F$$

- Effective input noise temperature:

S_{in} , N_{in} 

$S_{out} = G\, S_{in}$

$N_{out} = G\, N_{in} + N_{amp} = G\, k_B T_{in} + N_{amp}$

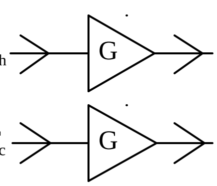
define N_{amp} / $G k_B T_e$

So $N_{out} = G k_B (T_{in} + T_e)$

$$F = \frac{Noise\ out}{Noise\ in} = \frac{G(T_0 + T_e)}{G T_0} \qquad F(dB) = 10 \log_{10}\left(\frac{T_0 + T_e}{T_0}\right)$$

Note: G, F, T_e all depend on ' $_{source}$.

Simple Case, all ' 's equal

T_h 

$N_{out,h} = G k_B (T_h + T_e)$

T_c

$N_{out,c} = G k_B (T_c + T_e)$

Combine & solve:

$$G = \frac{N_h - N_c}{k_B (T_h - T_c)} \qquad T_e = \frac{N_c T_h - N_h T_c}{N_h - N_c} = \frac{T_h - Y T_c}{Y - 1} \qquad \text{where } Y = N_h / N_c$$

$$F = \frac{T_e + T_0}{T_0} = 1 + \frac{T_h - Y T_c}{(Y-1) T_0} = \frac{ENR}{Y-1} - \left(\frac{Y}{Y-1}\right)\left(\frac{T_c - T_0}{T_0}\right) \approx \frac{ENR}{(Y-1)}$$

if T_c . T_0

(290 K Y - 63 °F)

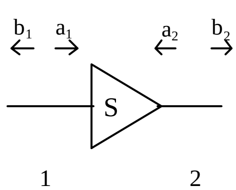
Noise Parameters

- But that's just for one value of Γ_{source}. Want to determine F or T_e for any Γ_{source}. So parameterize dependence on Γ_{source}.
- Several parameterizations in use; most common are variants of the IEEE [17] form. Particular IEEE form we use is [18]

$$T_e = T_{e,\min} + t\frac{|\Gamma_G - \Gamma_{opt}|^2}{(1 - |\Gamma_G|^2)\,|1 + \Gamma_{opt}|^2} \qquad t = 4\frac{R_n}{Z_0}$$

Wave Representation of Noise Matrix

- For microwave radiometry, wave representation [18-23] provides more flexibility.
- Linear 2-port:



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

- Noise matrix is defined by

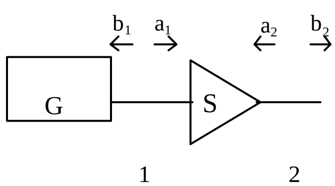
$$N_{ij} = \left\langle b_i b_j^* \right\rangle$$

or $\hat{N}_{ij} = \left\langle \hat{b}_i \hat{b}_j^* \right\rangle$ for intrinsic noise matrix

- Four real noise parameters:

$$<|\hat{b}_1|^2>, \; <|\hat{b}_2|^2>, \; <\hat{b}_1 \, \hat{b}_2^*>$$

- Output noise temperature T_2



$$k_B T_2 = \frac{|S_{21}|^2}{(1-|\Gamma_{GS}|^2)} \left[N_G + N_1 + N_2 + N_{12} \right]$$

$$N_G = \frac{(1-|\Gamma_G|^2)}{|1-\Gamma_G S_{11}|^2} T_G$$

$$N_1 = \left| \frac{\Gamma_G}{1-\Gamma_G S_{11}} \right|^2 <|\hat{b}_1|^2>$$

$$N_2 = <|\hat{b}_2/S_{21}|^2>$$

$$N_{12} = 2\,Re\left[\frac{\Gamma_G}{(1-\Gamma_G S_{11})} <\hat{b}_1 (\hat{b}_2/S_{21})^*> \right]$$

- So for T_e we have

$$T_e = \frac{|\Gamma_G|^2}{(1 - |\Gamma_G|^2)} X_1 + \frac{|1 - \Gamma_G S_{11}|^2}{(1 - |\Gamma_G|^2)} X_2 + \frac{2}{(1 - |\Gamma_G|^2)} Re[(1 - \Gamma_G S_{11})^* \Gamma_G X_{12}] .$$

where $X_1 \equiv \langle |\hat{b}_1|^2 \rangle$, $X_2 \equiv \langle |\hat{b}_2/S_{21}|^2 \rangle$, $X_{12} \equiv \langle \hat{b}_1 (\hat{b}_2/S_{21})^* \rangle$

- Whereas IEEE parameterization is

$$T_e = T_{e,\min} + t \frac{|\Gamma_G - \Gamma_{opt}|^2}{(1 - |\Gamma_G|^2)|1 + \Gamma_{opt}|^2}$$

- We can relate the two:

$$t = X_1 + |1 + S_{11}|^2 X_2 - 2\, Re[(1 + S_{11})^* X_{12}] ,$$

$$T_{e,\min} = \frac{X_2 - |\Gamma_{opt}|^2 [X_1 + |S_{11}|^2 X_2 - 2\, Re(S_{11}^* X_{12})]}{(1 + |\Gamma_{opt}|^2)} ,$$

$$\Gamma_{opt} = \frac{\eta}{2} \left(1 - \sqrt{1 - \frac{4}{|\eta|^2}} \right) ,$$

$$\eta = \frac{X_2 (1 + |S_{11}|^2) + X_1 - 2\, Re(S_{11}^* X_{12})}{(X_2 S_{11} - X_{12})} .$$

- Going the other way,

$$X_1 = T_{e,\min}(|S_{11}|^2 - 1) + \frac{t|1 - S_{11}\Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2},$$

$$X_2 = T_{e,\min} + \frac{t|\Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2},$$

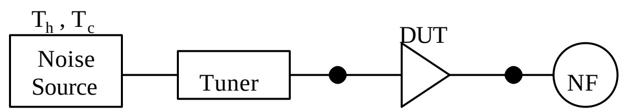
$$X_{12} = S_{11} T_{e,\min} - \frac{t\Gamma_{opt}^{*}(1 - S_{11}\Gamma_{opt})}{|1 + \Gamma_{opt}|^2}.$$

note bound implied by $X_1 > 0$.

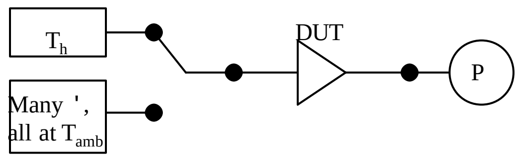
Measuring Noise Parameters

- Many different methods [18,20,22,24-34], most based on IEEE parameterization.
- Basic idea of (almost) all methods is to
  – present amplifier (or device) with a variety of different input terminations (Γ & T),
  – have an equation for the “output” in terms of the noise parameters and known quantities (Γ’s, T’s, S-parameters),
  – determine noise parameters by a fit to the measured output.
  – Need good distrib. of Γ’s in complex plane.

- “Output” can be
  - Noise figure [24]

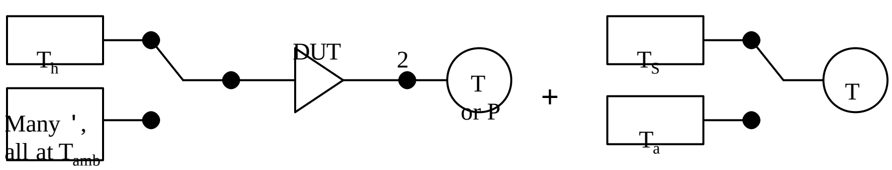


  - Power [25]



  - Note: output Γ, matching, available power, etc.

- Noise-matrix approach [22,23,30] to measuring noise parameters:



$$N_G = \frac{(1-|\Gamma_G|^2)}{|1-\Gamma_G S_{11}|^2} T_G$$

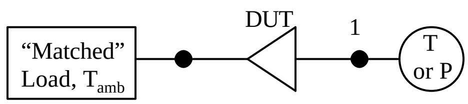
$$k_B T_2 = \frac{|S_{21}|^2}{(1-|\Gamma_{GS}|^2)} [N_G + N_1 + N_2 + N_{12}]$$

$$N_1 = \left|\frac{\Gamma_G}{1-\Gamma_G S_{11}}\right|^2 \langle|\hat{b}_1|^2\rangle$$

$$N_2 = \langle|\hat{b}_2/S_{21}|^2\rangle$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{\Gamma_G}{(1-\Gamma_G S_{11})}\langle\hat{b}_1(\hat{b}_2/S_{21})^*\rangle\right]$$

- Supplemental measurement (noise matrix) [27,31]



$$k_B T_1 = \frac{1}{\left(1-\left|\Gamma'_{GS}\right|^2\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$\Gamma'_{GS} = S_{11} + \frac{\Gamma_G S_{12} S_{21}}{\left(1-\Gamma_G S_{22}\right)}$$

$$N_G = \frac{\left|S_{12}\right|^2\left(1-\left|\Gamma_G\right|^2\right)}{\left|1-\Gamma_G S_{22}\right|^2} T_G$$

$$N_1 = \left\langle\left|\hat{b}_1\right|^2\right\rangle$$

$$N_2 = \left|\frac{S_{12} S_{21}\Gamma_G}{1-\Gamma_G S_{22}}\right|^2 \left\langle\left|\hat{b}_2 / S_{21}\right|^2\right\rangle$$

$$N_{12} = 2\mathrm{Re}\left\{\frac{S_{12} S_{21}\Gamma_G}{\left(1-\Gamma_G S_{22}\right)}\left\langle \hat{b}_1^*\left(\hat{b}_2 / S_{21}\right)\right\rangle\right\}$$

- Noise-Parameter Uncertainties
  - Monte Carlo method is probably the most practical [26,35-38]
  - Some general approximate features [38]:
    - Uncerts in *G* and T_{min} (& F_{min}) are dominated by uncert in T_h. 0.1 dB uncert in T_h Y - 0.1 dB uncert in *G* and F_{min}.
    - Uncerts in $'_{opt}$ are dominated by uncerts in $'_G$'s. Uncert in Re or Im $'_{opt}$ is - 3 or 4H uncert in Re or Im $'_G$ (for 13 terminations).
    - *t* is sensitive to just about everything.
    - T_{amb} is not a major factor, because it is much better known than T_h. Note, however, that it could affect T_h or the amplifier properties.

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